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Magnetic and Dyonic Black Holes In D=4 Gauged Supergravity

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Abstract

Magnetic and Dyonic solutions are constructed for the theory of abelian gauged $N = 2$ gauged four dimensional supergravity coupled to vector multiplets. The solutions found preserve 1/4 of the supersymmetry.

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1 Introduction

There has been lots of interest in the study of solutions of gauged supergravity theories in recent years [1, 2, 3, 4, 5, 6, 7, 8, 9]. Anti- de Sitter (AdS) black holes solutions and their supersymmetric properties, in the context of gauged $N = 2$, $D = 4$ supergravity theory, were first considered in [10, 11]. One of the motivations for the renewed interest is the fact that the ground state of gauged supergravity theories is AdS space-time and therefore, solutions of these theories may have implications for the AdS/conformal field theory correspondence proposed in [12]. The classical supergravity solution can provide some important information on the dual gauge theory in the large N (the rank of the gauge group). An example of this is the Hawking-Page phase transition [13, 14]. Also, the proposed AdS/CFT equivalence opens the possibility for a microscopic understanding of the Bekenstein-Hawking entropy of asymptotically anti-de Sitter black holes [15, 16]. Moreover, AdS spaces are known to admit "topological" black holes with some unusual geometric and physical properties (see for example [17]).

Of particular interest in this context are black objects in AdS space which preserve some fraction of supersymmetry. On the CFT side, these supergravity vacua could correspond to an expansion around non-zero vacuum expectation values of certain operators.

Although lots is known about solutions in ungauged supergravity theories [18], the situation is different for the gauged cases. Our main purpose in this paper is to obtain supersymmetric solutions for a very large class of supergravity theories coupled to vector multiplets. The theories we will consider are four dimensional abelian gauged $N = 2$ supergravity theories coupled to vector multiplets. Electrically charged solutions of these theories, asymptotic to AdS_4 space-time, were constructed in [7]. In this paper we will be mainly concerned with the construction of magnetic as well as dyonic solutions.

Our work in this paper is organized as follows. Section two contains a brief review of $D = 4$, $N = 2$ gauged supergravity where we collect formulae of special geometry [20] necessary for our discussion and for completeness we present the electrically charged solutions of [7]. In section three, supersymmetric magnetic and dyonic solutions are constructed together with their Killing spinors. Finally, our results are summarized and discussed.

2 Gauged Supergravity and Special Geometry

The construction of BPS solutions of the theory of ungauged and gauged $N = 2$ supergravity coupled to vector supermultiplets, relies very much on the structure of special geometry underlying these theories. The complex scalars z^A of the vector supermultiplets of the $N = 2$ supergravity theory are coordinates which parametrize a special Kähler manifold. The Abelian gauging is achieved by introducing a linear combination of the abelian vector fields A_μ^I of the theory, $A_\mu = \kappa_I A_\mu^I$ with a coupling constant g, where κ_I are constants. The couplings of the fermi-fields to this vector field break supersymmetry which in order to maintain one has to introduce gauge-invariant g-dependent terms. In a bosonic background, these additional terms produce a scalar potential [19]

$$V = g^2 \left(g^{A\bar{B}} \kappa_I \kappa_J f_A^I \bar{f}_{\bar{B}}^J - 3\kappa_I \kappa_J \bar{L}^I L^J \right). \tag{1}$$

The meaning of the various quantities in (1) is as follows. Roughly one defines a special Kähler manifold in terms of a flat $(2n + 2)-$ dimensional symplectic bundle over the Kähler-Hodge manifold, with the covariantly holomorphic sections

$$V = \begin{pmatrix} L^I \\ M_I \end{pmatrix}, \quad I = 0, \cdots, n. \qquad D_{\bar{A}} V = (\partial_{\bar{A}} - \frac{1}{2}\partial_{\bar{A}} K)V = 0,$$

obeying the symplectic constraint

$$i\langle V|\bar{V}\rangle = i(\bar{L}^I M_I - L^I \bar{M}_I) = 1. \tag{2}$$

The scalar metric appearing in the potential can then be expressed as follows

$$g_{A\bar{B}} = -i\langle U_A|\bar{U}_{\bar{B}}\rangle$$

where

$$U_A = D_A V = (\partial_A + \frac{1}{2}\partial_A K)V \quad = \begin{pmatrix} f_A^I \\ h_{AI} \end{pmatrix}.$$

In general, one writes $M_I = \mathcal{N}_{IJ} L^J$, $h_{AI} = \bar{\mathcal{N}}_{IJ} f_A^J$ and the metric can then be expressed as $g_{A\bar{B}} = -2(\mathrm{Im}\,\mathcal{N})_{IJ} f_A^I \bar{f}_{\bar{B}}^J$. Moreover, special geometry implies the following useful relation

$$g^{A\bar{B}} f_A^I \bar{f}_{\bar{B}}^J = -\frac{1}{2}(\mathrm{Im}\,\mathcal{N})^{IJ} - \bar{L}^I L^J. \tag{3}$$

The supersymmetry transformations for the gauginos and the gravitino in a bosonic background

$$\begin{aligned}
\delta\,\psi_\mu &= \left(\mathcal{D}_\mu + \frac{i}{4} T_{ab}\gamma^{ab}\gamma_\mu - ig\kappa_I A^I_\mu + \frac{i}{2} g\kappa_I L^I \gamma_\mu\right)\epsilon, \\
\delta\lambda^A &= \left(i\,\gamma^\mu \partial_\mu z^A + i\mathcal{G}^A_{\rho\sigma}\gamma^\rho\gamma^\sigma - g g^{A\bar{B}}\kappa_I f^I_{\bar{B}}\right)\epsilon.
\end{aligned} \tag{4}$$

where $\mathcal{D}_\mu$ is the covariant derivative, $\mathcal{D}_\mu = (\partial_\mu - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + \frac{i}{2}\, Q_\mu)$, where ω_μ^{ab} is the spin connection and Q_μ is the Kähler connection, which locally can be represented by

$$Q = -\frac{i}{2}\left(\partial_A K dz^A - \partial_{\bar{A}} K d\bar{z}^A\right). \tag{5}$$

The quantities $T_{\mu\nu}$ and $\mathcal{G}^A_{\rho\sigma}$ are given by

$$\begin{aligned}
T_{\mu\nu} &= 2i(\mathrm{Im}\mathcal{N}_{IJ}) L^I F^J_{\mu\nu} \\
\mathcal{G}^A_{\rho\nu} &= -g^{A\bar{B}}\, \bar{f}^I_{\bar{B}}\, (\mathrm{Im}\mathcal{N}_{IJ})\, F^{\;J}_{\rho\nu},
\end{aligned} \tag{6}$$

Electrically charged BPS solutions for the $N = 2$, $D = 4$ supergravity with vector multiplets were constructed in [7] and are given by

$$\begin{aligned}
& ds^2 = -\left(e^{2U} + g^2 r^2 e^{-2U}\right) dt^2 + \frac{1}{(e^{2U} + g^2 r^2 e^{-2U})} dr^2 + e^{-2U} r^2 (d\theta^2 + \sin^2\theta d\phi^2), \\
& e^{-2U} = Y^I H_I, \\
& i(\mathcal{F}_I(Y) - \bar{\mathcal{F}}_I(\bar{Y}) = H_I, \qquad Y^I = \bar{Y}^I \\
& A^I_t = e^{2U} Y^I.
\end{aligned} \tag{7}$$

where $H_I = \kappa_I + \frac{q_I}{r}$, and q_I are the electric charges. These electric BPS solutions break half of supersymmetry where the Killing spinor ϵ satisfies

$$\epsilon = (a\gamma_0 + b\gamma_1)\epsilon. \tag{8}$$

where

$$a = \frac{1}{\sqrt{1+g^2r^2e^{-4U}}},$$
$$b = -i\frac{gre^{-2U}}{\sqrt{1+g^2r^2e^{-4U}}}, \tag{9}$$

The solution for the Killing spinor is given by

$$\epsilon(r) = \frac{1}{2\sqrt{gr}}e^{\frac{igt}{2}}e^{-\frac{1}{2}\gamma_0\gamma_1\gamma_2\theta}e^{-\frac{1}{2}\gamma_2\gamma_3\phi}e^{U+T}\left(\sqrt{f-1}-i\gamma_1\sqrt{f+1}\right)(1-\gamma_0)\epsilon_0 \tag{10}$$

where ϵ_0 is an arbitrary constant spinor and $T = \int^r \frac{1}{2r'}(1 - r'\partial_{r'}U(r'))dr'$.

3 Supersymmetric solutions

In this section, we find magnetic and dyonic BPS solutions (with constant scalars) of the theory of abelian gauged $N = 2$ supergravity coupled to vector supermultiplets. The solutions found preserve a quarter of supersymmetry. We consider the following general form for the metric

$$ds^2 = -e^{2A}dt^2 + e^{2B}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{11}$$

The vielbeins of this metric can be taken as

$$e_t^0 = e^A, \quad e_r^1 = e^B, \quad e_\theta^2 = r, \quad e_\phi^3 = r\sin\theta,$$
$$e_0^t = e^{-A}, \quad e_1^r = e^{-B}, \quad e_2^\theta = \frac{1}{r}\ , \quad e_3^\phi = \frac{1}{r\sin\theta}. \tag{12}$$

and the spin connections for the above metric are

$$\omega_t^{01} = A'e^{A-B},$$
$$\omega_\theta^{12} = -e^{-B},$$
$$\omega_\phi^{13} = -e^{-B}\sin\theta,$$
$$\omega_\phi^{23} = -\cos\theta. \tag{13}$$

3.1 Magnetic Solutions

First we concentrate on the purely magnetic BPS solutions. Therefore, we take the vector potential to have only one non-vanishing component, i. e.,

$$A^I_\phi = q^I \cos\theta, \qquad F^I_{\theta\phi} = -q^I \sin\theta, \tag{14}$$

Using the above ansatz for the gauge fields and the spin connections in (13), the supersymmetry transformations for the gravitino (4) give

$$\begin{aligned}
\delta\psi_t &= \left[\partial_t - \frac{1}{2}A' e^{A-B}\gamma_0\gamma_1 + \frac{i}{2}e^A T_{23}\gamma_2\gamma_3\gamma_0 + \frac{i}{2}g e^A \kappa_I L^I \gamma_0\right]\epsilon, \\
\delta\psi_\theta &= \left[\partial_\theta + \frac{1}{2}e^{-B}\gamma_1\gamma_2 + \frac{i}{2}T_{23} r\gamma_3 + \frac{i}{2}g r\kappa_I L^I \gamma_2\right]\epsilon, \\
\delta\psi_r &= \left[\partial_r + \frac{i}{2}T_{23}\gamma_2\gamma_3\gamma_1 e^B + \frac{i}{2}g(\kappa_I L^I)\gamma_1 e^B\right]\epsilon, \\
\delta\psi_\phi &= [\partial_\phi + \frac{1}{2}\cos\theta\gamma_2\gamma_3 + \frac{1}{2}e^{-B}\sin\theta\gamma_1\gamma_3 + r\sin\theta(-T_{23}\gamma_2 + \frac{i}{2}g\kappa_I L^I\gamma_3) - ig\kappa_I A^I_\phi]\epsilon.
\end{aligned} \tag{15}$$

and for purely magnetic solutions we have

$$T_{23} = 2i(\mathrm{Im}\mathcal{N}_{IJ})L^I F^J_{23}.$$

We will find supersymmetric configuration admitting Killing spinors which satisfy the following conditions

$$\begin{aligned}
\gamma_1\epsilon &= i\epsilon, \\
\gamma_2\gamma_3\epsilon &= i\epsilon.
\end{aligned} \tag{16}$$

Because of the double projection on the spinor ϵ, it is independent of the angular variables θ and ϕ. With the conditions (16), one obtains from the vanishing of the gravitino supersymmetry transformations in (15) the following equations

$$\begin{aligned}
-\frac{1}{2}A' e^{-B} + \frac{i}{2}T_{23} + \frac{1}{2}g(\kappa_I L^I) &= 0, \\
-\frac{1}{2}e^{-B} - \frac{i}{2}T_{23} r + \frac{1}{2}g(\kappa_I L^I) r &= 0, \\
\frac{1}{2}\cos\theta - g(\kappa_I A^I_\phi) &= 0.
\end{aligned} \tag{17}$$

and

$$\begin{aligned}\partial_t\epsilon &= 0,\\ \partial_\theta\epsilon &= 0,\\ \left[\partial_r - \frac{i}{2}T_{23}e^B - \frac{1}{2}g(\kappa_I L^I)\gamma_1 e^B\right]\epsilon &= 0,\\ \partial_\phi\epsilon &= 0.\end{aligned} \tag{18}$$

If one takes the ansatz

$$A = -B,$$

then from the first two equations in (17) one obtains

$$\begin{aligned}\partial\left(e^{-B}\right) &= iT_{23} + g(\kappa_I L^I),\\ e^{-B} &= -iT_{23}r + gr(\kappa_I L^I).\end{aligned} \tag{19}$$

An obvious solution to the above equations can be obtained if we set $(\kappa_I L^I)$ to a constant say $\kappa_I L^I$ =1. Moreover, from the third relation in (17), one gets the following "quantization relation"

$$\kappa_I q^I = \frac{1}{2g}.$$

Therefore, as a solution for the holomorphic sections we take real L^I and imaginary M_I, with

$$L^I = 2g\ q^I. \tag{20}$$

Using the symplectic constraint (2),

$$i(\bar{L}^I M_I - L^I\bar{M}_I) = iL^I(M_I - \bar{M}_I) = 2iL^I M_I = 1. \tag{21}$$

Then this relation together with (20) implies that the magnetic central charge $Z_m = M_I q^I = -\frac{i}{4g}$, and therefore we obtain

$$T_{23} = 2i(\mathrm{Im}\mathcal{N}_{IJ})L^I F^J_{23} = -2\frac{M_I q^I}{r^2} = \frac{i}{2gr^2},$$

and from (19) we obtain the following solution

$$e^{-B} = -iT_{23}r + gr(\kappa_I L^I) = gr + \frac{1}{2gr}.$$

Finally one has to check for the vanishing of the our the gaugino supersymmetry transformation for our solution. The gaugino transformation is given by

$$\delta\lambda^A = \left(i\,\gamma^\mu\partial_\mu z^A + i\mathcal{G}^A_{\rho\sigma}\gamma^\rho\gamma^\sigma - gg^{A\bar{B}}\kappa_I f^I_{\bar{B}}\right)\epsilon \tag{22}$$

where $\mathcal{G}^A_{\rho\nu} = -g^{A\bar{B}}\,\bar{f}^I_{\bar{B}}\,(\mathrm{Im}\mathcal{N}_{IJ})\,F^{\ J}_{\rho\nu}$, $g^{A\bar{B}}$ is the inverse Kähler metric and $\bar{f}^I_{\bar{B}} = (\partial_{\bar{B}} + \frac{1}{2}\partial_{\bar{B}}K)\bar{L}^I$. To demonstrate the vanishing of the gaugino supersymmetry variations for the choice of ϵ, it is more convenient to multiply with f^I_A. This gives using our solution and the relations following from special geometry (3),

$$f^I_A\delta\lambda^{\alpha A} = \left(i\gamma^\mu\partial_\mu z^A(\partial_A + \frac{1}{2}\,\partial_A K)L^I + \frac{i}{2}(F^{\,I}_{\mu\nu} - iL^I T_{\mu\nu})\gamma^\mu\gamma^\nu + \frac{g}{2}\mathrm{Im}\mathcal{N}^{IJ}\kappa_J + g\kappa_J L^J L^I\right)\epsilon$$

The above transformation vanishes provided

$$\begin{aligned} (F^I_{23} - iL^I T_{23}) &= 0, \\ \frac{g}{2}\mathrm{Im}\mathcal{N}^{IJ}\kappa_J + g\kappa_J L^J L^I &= 0. \end{aligned} \tag{23}$$

Using our solution it can be easily seen that the above equations are satisfied.

In summary, we have obtained a BPS magnetic solution preserving a quarter of supersymmetry for the theories of abelian gauged $N = 2$, $D = 4$ supergravity theories with vector multiplets. This solution is given by

$$\begin{aligned} ds^2 &= -(\frac{1}{2gr} + gr)^2 dt^2 + (\frac{1}{2gr} + gr)^{-2} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \\ L^I &= 2g\ q^I, \qquad A^I_\phi = q^I\cos\theta. \end{aligned}$$

3.2 Dyonic Solutions

In this section we generalize the previous discussion to include electric charges and thus obtaining dyonic solutions. From the outset we set $A = -B$ in the ansatz (11) as well as the condition $\kappa_I L^I = 1$. The supersymmetric transformation for the gravitino in this case gives

$$\begin{aligned}
\delta\psi_t &= \left[\partial_t + \frac{1}{2}B' e^{-2B}\gamma_0\gamma_1 + \frac{i}{2}T_{23}\gamma_2\gamma_3\gamma_0 e^{-B} + \frac{i}{2}T_{01}\gamma_1 e^{-B} - ig\kappa_I A^I_t + \frac{i}{2}g e^{-B}\gamma_0\right]\epsilon,\\
\delta\psi_\theta &= \left[\partial_\theta + \frac{1}{2}e^{-B}\gamma_1\gamma_2 + \frac{i}{2}T_{23} r\gamma_3 - \frac{i}{2}T_{01} r\gamma_0\gamma_1\gamma_2 + \frac{i}{2}g r\gamma_2\right]\epsilon,\\
\delta\psi_r &= \left[\partial_r + \frac{i}{2}T_{23}\gamma_2\gamma_3\gamma_1 e^{B} + \frac{i}{2}T_{01}\gamma_0 e^{B} + \frac{i}{2}g\gamma_1 e^{B}\right]\epsilon,\\
\delta\psi_\phi &= [\partial_\phi + \frac{1}{2}\cos\theta\gamma_2\gamma_3 + \frac{1}{2}e^{-B}\sin\theta\gamma_1\gamma_3\\
&\quad + r\sin\theta(\frac{i}{2}T_{23}\gamma_2\gamma_3 - \frac{i}{2}T_{01}\gamma_0\gamma_1)\gamma_3 + \frac{i}{2}g\gamma_3 - ig\kappa_I A^I_\phi]\epsilon,
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
T_{23} &= 2i\,\mathrm{Im}\,\mathcal{N}_{IJ}L^I F^J_{23},\\
T_{01} &= 2i\,\mathrm{Im}\,\mathcal{N}_{IJ}L^I F^J_{01}.
\end{aligned}$$

The dyonic solutions can now be easily obtained by modifying one of the supersymmetry breaking conditions (16) and imposing the following conditions on the Killing spinors

$$\begin{aligned}
(ia\gamma_0 + b\gamma_1)\epsilon &= i\epsilon,\\
\gamma_2\gamma_3\epsilon &= i\epsilon.
\end{aligned} \tag{25}$$

clearly the coefficients a and b must satisfy the condition $a^2 + b^2 = 1$.

Using the conditions (25), then from the vanishing of the transformations (24) we obtain the equations

$$
\begin{aligned}
-\frac{1}{2b}e^{-B} - \frac{i}{2}T_{23}r + \frac{i}{2}T_{01}\frac{a}{b}r + \frac{1}{2}gr = 0,\\
\frac{a}{2b}e^{-B} - \frac{i}{2}T_{01}r\frac{1}{b} = 0,\\
\frac{1}{2b}B'e^{-B} + \frac{i}{2}T_{23} - \frac{i}{2}T_{01}\frac{a}{b} + \frac{1}{2}g = 0,\\
-\frac{a}{2b}B'e^{-2B} - g\kappa_I A_t^I + \frac{i}{2}T_{01}\frac{1}{b}e^{-B} = 0.
\end{aligned}
\tag{26}
$$

together with

$$
\begin{aligned}
\partial_t\epsilon = 0,\\
\partial_\theta\epsilon = 0,\\
\left[\partial_r - \frac{1}{2}T_{23}\gamma_1 e^B + \frac{i}{2}T_{01}\gamma_0 e^B + \frac{i}{2}g\gamma_1 e^B\right]\epsilon = 0,\\
\partial_\phi\epsilon = 0.
\end{aligned}
\tag{27}
$$

As an ansatz for the gauge and scalar fields we take

$$
A_t^I = \frac{Z_e L^I}{r}, \qquad A_\phi^I = q^I\cos\theta, \qquad F_{01}^I = \frac{Z_e L^I}{r^2}, \qquad F_{23}^I = -\frac{q^I}{r^2}, \qquad L^I = 2gq^I,
$$

where $Z_e = L^I Q_I$, is the electric central charge, Q_I being the electric charge. Therefore we obtain

$$
\begin{aligned}
T_{23} &= 2i\,\mathrm{Im}\mathcal{N}_{IJ}L^I F_{23}^J = -2\frac{Z_m}{r^2} = \frac{i}{2gr^2},\\
T_{01} &= 2i\,\mathrm{Im}\mathcal{N}_{IJ}L^I F_{01}^J = -i\frac{Z_e}{r^2}.
\end{aligned}
$$

From (26) we obtain the following solution for the metric and the functions a and b,

$$
\begin{aligned}
e^{-2B} &= J_1^2 + J_2^2. \\
J_1 &= iT_{01}r = \kappa_I A_t^I = \frac{Z_e}{r}. \\
J_2 &= gr - iT_{23}r = gr + 2i\frac{Z_m}{r} = gr + \frac{1}{2gr} \\
a &= J_1 e^B, \quad b = J_2 e^B
\end{aligned} \tag{28}
$$

From the vanishing of the gaugino transformations in the presence of electric charges, we get the conditions (23) together with the newcondition

$$
(F_{01}^I - iL^I T_{01}) = 0.
$$

All these conditions can be seen to be satisfied for our ansatz.

Finally, we summarize dyonic solutions of $N = 2$, $D = 4$ gauged supergravity with vector multiplets by

$$
\begin{aligned}
ds^2 &= -\left((gr + \frac{1}{2gr})^2 + \frac{Z_e^2}{r^2}\right) dt^2 + \left((gr + \frac{1}{2gr})^2 + \frac{Z_e^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \\
L^I &= 2gq^I, \quad A_\phi = q^I \cos\theta, \quad Z_e = L^I Q_I.
\end{aligned}
$$

3.3 Solutions With Hyperbolic And Flat Transverse Spaces

Clearly the spherical solutions we found represent naked singularities. However, one can obtain extremal purely magnetic solutions with event horizons if the two sphere is replaced with the quotients of the hyperbolic two-space H^2. In this case, it can be shown that one obtains the following solitonic dyonic solution

$$
\begin{aligned}
ds^2 &= -\left(gr - \frac{1}{2gr}\right)^2 dt^2 + \left(gr - \frac{1}{2gr}\right)^{-2} dr^2 + r^2(d\theta^2 + \sinh^2\theta d\phi^2). \\
A_\phi^I &= q^I \cosh\theta, \quad L^I = 2gq^I.
\end{aligned}
$$

As the Killing spinors do not depend on the coordinates θ, ϕ of the transverse hyperbolic space, one could also compactify the H^2 to a Riemann surface $\mathcal{S}_n$ of genus n, and the resulting solution would still preserve one quarter of supersymmetry.
Whereas the spherical magnetic solution contains a naked singularity, the hyperbolic magnetic black hole solution is a genuine black hole solution which has an event horizon at $r = r_+ = 1/(g\sqrt{2})$. In the near horizon region, the metric reduces to the product manifold $AdS_2 \times H^2$ and thus supersymmetry is enhanced [21].

One can also consider flat transverse space with vanishing gauge fields. Clearly one finds the solution which is locally AdS_4. However, one may wish to compactify the (θ, ϕ) sector to a cylinder or a torus, considering thus a quotient space of AdS_4. This identification results in AdS_4 quotient space preserving half of the supersymmetries.

3.4 Killing spinors

We now derive an expression for the Killing spinors of our solutions. From (27), one obtain the following equations for the Killing spinors

$$
\begin{aligned}
\partial_t \epsilon &= 0, \\
\partial_\theta \epsilon &= 0, \\
\left(\partial_r + \frac{1}{2r} + ig\gamma_1 e^B\right)\epsilon &= 0, \\
\partial_\phi \epsilon &= 0.
\end{aligned}
\tag{29}
$$

Using the method of [10], one obtain the following solution for the Killing spinor

$$
\epsilon(r) = \left(\sqrt{e^{-B} + gr + \frac{1}{2gr}} - \gamma_0 \sqrt{e^{-B} - gr - \frac{1}{2gr}}\right) P(-i\gamma_1) P(-i\gamma_{23}) \epsilon_0
$$

where ϵ_0 is a constant spinor and where we have used the notation $P(\Gamma) = \frac{1}{2}(1 + \Gamma)$, where Γ is an operator satisfying $\Gamma^2 = 1$.

For the purely magnetic solution, i. e, $Q_I = 0$, the Killing spinor reduces to the simple form

$$\epsilon(r) = \sqrt{gr + \frac{1}{2gr}} P(-i\gamma_1) P(-i\gamma_{23}) \epsilon_0$$

It must be mentioned as was observed in [10] that for the purely magnetic solution , e^{-B} and $\epsilon(r)$ are invariant under

$$r \longrightarrow \frac{1}{2g^2 r}$$

and under such a transformation, the spatial component of the metric is conformally rescaled. Clearly the Killing spinors for the hyperbolic case are similar and can be obtained by replacing $gr + \frac{1}{2gr}$ by $gr - \frac{1}{2gr}$ everywhere[2].

4 Discussion

In summary, we have obtained magnetic and dyonic BPS solutions of the theory of abelian gauged $N = 2$ supergravity coupled to a number of vector supemultiplets. These solutions preserve a quarter of supersymmetry. Due to the quantization of the magnetic central charge for these solutions, the metric of the magnetic solutions takes a universal form for any choice of the prepotential and for any number of vector multiplets (charges). The dyonic solutions depend also on the electric central charge. We note that our solution is expressed in terms of the holomorphic sections and the central charges of the theory and therefore independent of the existence of a holomorphic prepotential. A subclass of solutions of $N = 2$ supergravity (for a particular choice of prepotential) are actually also solutions of supergravity theories with more supersymmetry ,(i.e. $N = 4$ or $N = 8$ supersymmetries). The spherically symmetric solutions has the common feature of representing naked singularities. However, it is known from the work of [11] that the spherical Kerr-Newman-AdS solution is both supersymmetric and extreme. This means that one can obtain supersymmetric extremal black holes in an AdS background if the black holes are rotating. Thus the situation in anti-de Sitter background seems to be the opposite of that of the asymptotically flat.

Whereas the spherical BPS magnetic black holes contain a naked singularity, the hyperbolic black holes have an event horizon. Supersymmetric

higher genus solution black holes with magnetic charges preserve the same amount of supersymmetry which is enhanced near the horizon as has been demonstrated in [21] for the pure supergravity theory without vector multiplets. Here we found that the situation remains the same in the presence of vector multiplets. In summary, in order to get genuine black holes for the theories we have considered in this paper, one should allow for rotations as well as nonspherical symmetry. This will be reported on in a future publication.

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